4

MEASUREMENT AND GEOMETRY

GEOMETRY

Geometry is all around us. Designs and patterns involving angles, parallel lines, triangles and quadrilaterals can be found everywhere, in our homes, on transport, in construction, art and nature. This scene from the National Museum of Australia in Canberra shows the importance of angles, lines and shapes in architecture and design.

Chapter outline

	Working mathematically				
4.01 Angle geometry	U	F		R	C
4.02 Angles on parallel lines	U	F		R	C
4.03 Line and rotational symmetry	U	F		R	
4.04 Classifying triangles	U	F		R	C
4.05 Classifying quadrilaterals	U	F		R	C
4.06 Properties of quadrilaterals	U	F		R	C
4.07 Angle sums of triangles and quadrilaterals	U	F		R	C
4.08 Extension: Angle sum of a polygon*#	U	F		R	C

***Year 9 Extension, Stage 5.2**

#NSW ONLY, NOT IN AUSTRALIAN CURRICULUM

Wordbank

angle sum The total of the sizes of the angles in a shape, such as a triangle

bisect To cut in half

convex quadrilateral A quadrilateral whose vertices all point outwards.

diagonal An interval joining 2 non-adjacent vertices of a shape

exterior angle of a triangle An 'outside' angle of a triangle formed after extending one of the sides of the triangle

scalene triangle A triangle with no equal sides

supplementary angles 2 angles that add to 180°

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- solve geometrical problems involving angles on a straight line, angles at a point and vertically opposite angles
- solve geometrical problems involving corresponding, alternate and co-interior angles on parallel lines crossed by a transversal
- identify line and rotational symmetries
- classify triangles according to their side and angle properties
- identify convex and non-convex quadrilaterals
- classify squares, rectangles, rhombuses, parallelograms, kites and trapeziums
- solve geometrical problems involving the angle sum of a triangle and quadrilateral and the exterior angle of a triangle
- (EXTENSION, STAGE 5.2) solve geometrical problems involving the angle sum of a polygon

SkillCheck ANSWERS ON P. 553

Types of angles

A page of angles

Angle cards

1 Draw 2 different examples of each type of angle.

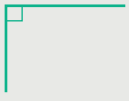
a acute angle **b** right angle **c** obtuse angle **d** reflex angle

2 Classify each angle size as being acute, obtuse, right, reflex, straight or a revolution.

a 25° **b** 100° **c** 300° **d** 128°

e 90° **f** 360° **g** 286° **h** 180°

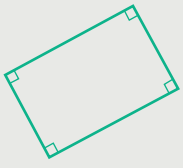
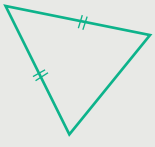
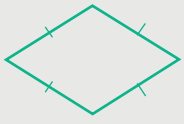
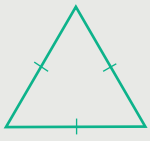
3 Name each type of angle(s) marked.

a  **b**  **c** 

4 Find the value of x in each equation.

a $x + 30 = 90$ **b** $x + 57 = 180$ **c** $x + 121 + 77 = 360$

5 Copy each shape, name it and draw all axes of symmetry.

a  **b**  **c**  **d** 

6 Write the order of rotational symmetry of each shape in question **5**.

7 Which quadrilateral has all 4 sides equal and all 4 angles equal?
Select the correct answer **A**, **B**, **C** or **D**.

A parallelogram **B** rhombus **C** rectangle **D** square

8 Draw a scalene triangle.

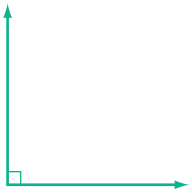
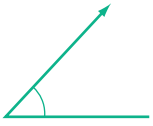
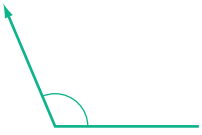
9 **a** Draw a parallelogram and its diagonals.

b Are the lengths of the diagonals of a parallelogram equal?

Angle geometry

4.01

Classifying angles

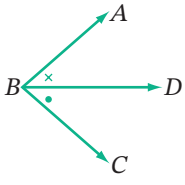
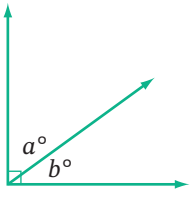
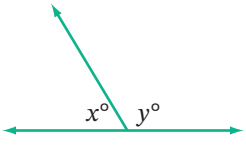
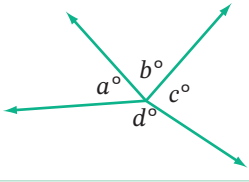
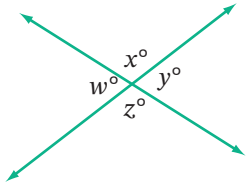
Right angle	Straight angle	Revolution
90° (quarter-turn)	180° (half-turn)	360° (complete turn)
		
Acute angle	**Obtuse angle**	**Reflex angle**
Less than 90°	Between 90° and 180°	Between 180° and 360°
		

Straight angles, right angles and revolutions

Angles: A dog day

Angles

Angle facts

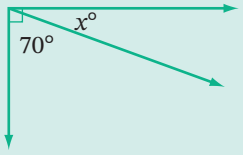
Adjacent angles	Complementary angles	Angles in a right angle
Angles next to each other, sharing a common arm. $\angle ABD$ and $\angle DBC$ are adjacent.	2 angles that have a sum of 90°, for example, 35° and 55°.	Are complementary. $a + b = 90$
	**Supplementary angles** 2 angles that have a sum of 180°, for example, 140° and 40°.	
Angles on a straight line	**Angles at a point**	**Vertically opposite angles**
Are supplementary. $x + y = 180$	(In a revolution) Add up up to 360°. $a + b + c + d = 360$	Are equal. $w = y$ and $x = z$
		

Angles at a point and vertically opposite angles

Example 1

Find the value of each variable, giving reasons.

a



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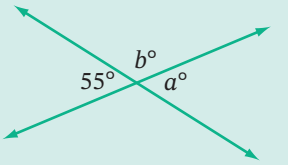

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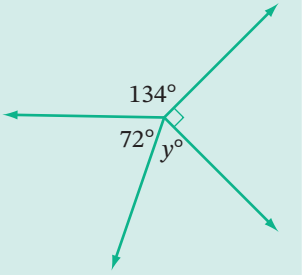


Solution

a $x + 70 = 90$ (Angles in a right angle)

$x = 90 - 70$

$= 20$

b $a = 55$ (Vertically opposite angles.)

$b + 55 = 180$ (Angles in a straight line)

$b = 180 - 55$

$= 125$

c $y + 72 + 134 + 90 = 360$ (Angles at a point)

$y + 296 = 360$

$y = 360 - 296$

$= 64$

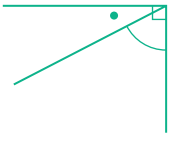
The reason is written inside brackets.

EXERCISE 4.01 ANSWERS ON P. 553

Angle geometry UFRC

1 Classify each type of angle(s). C

a



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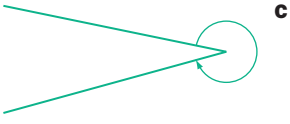

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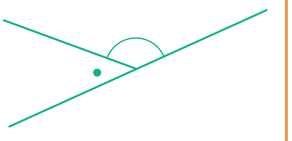

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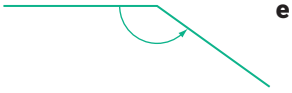

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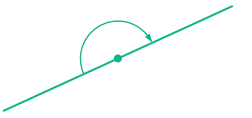

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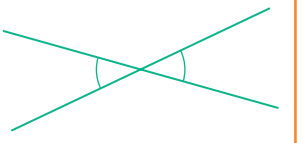

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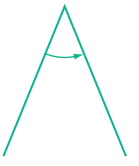

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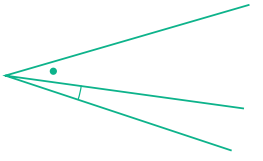

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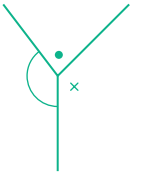


2 For each angle size, find the complementary angle.

a 16° **b** 33° **c** 71° **d** 2°

3 For each angle size, find the supplementary angle.

a 25° **b** 149° **c** 107° **d** 48°

4 What is the sum of the angles at a point? Select the correct answer **A**, **B**, **C** or **D**.

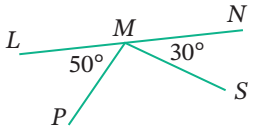
A 90° **B** 180° **C** 270° **D** 360°

5 Find size of $\angle SMP$. Select **A**, **B**, **C** or **D**.

A 80° **B** 280°

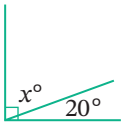
C 100° **D** 10°

EXAMPLE 1



6 Find the value of each variable, giving reasons. R C

a



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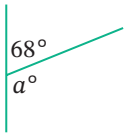

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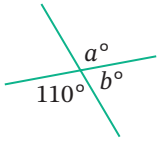

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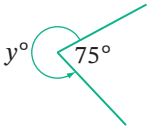

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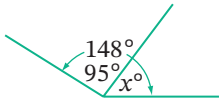

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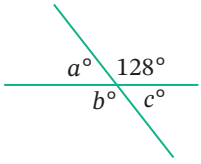

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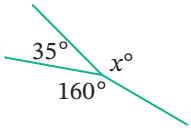

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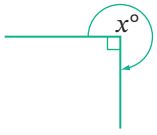

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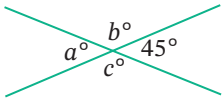

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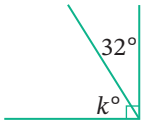

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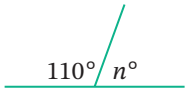

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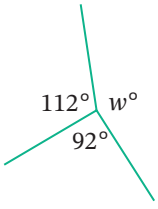

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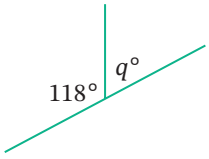

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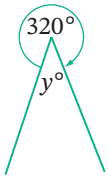

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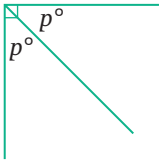

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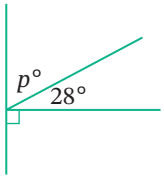

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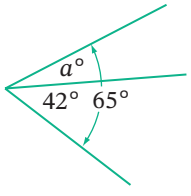

Foundation Standard Complex

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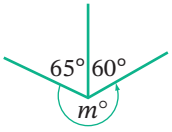
7 Find the value of each variable, giving reasons. R C

a



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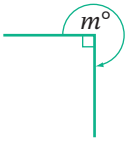

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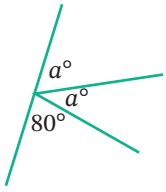

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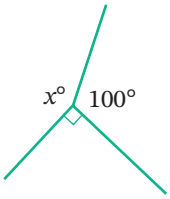

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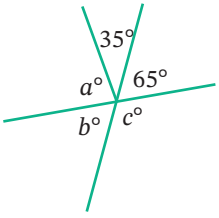

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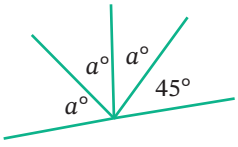

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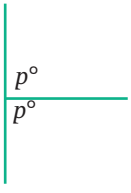

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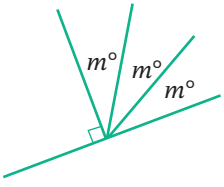
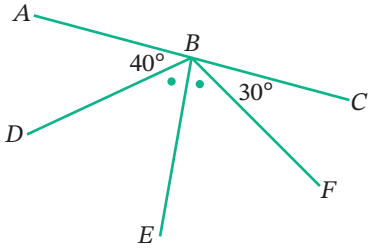


8 *ABC* is a straight line and *EB* bisects ∠*DBF*. Find the size of ∠*EBC*.
Select **A**, **B**, **C** or **D**. R

A 95°
B 110°
C 85°
D 140°



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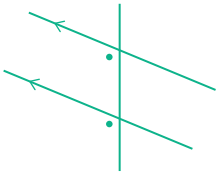
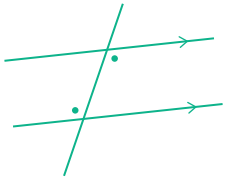
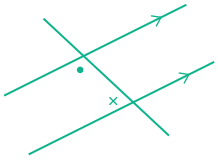
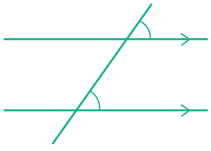
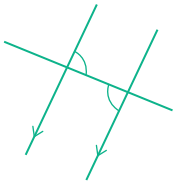
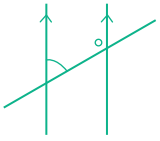
Angles on parallel lines

4.02

When **parallel lines** are crossed by another line (called a **transversal**), special pairs of angles are formed.

Corresponding angles	Alternate angles	Co-interior angles
Corresponding angles on parallel lines are equal.	Alternate angles on parallel lines are equal.	Co-interior angles on parallel lines are supplementary (add to 180°).
		
		

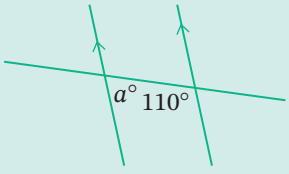
- **Corresponding angles** are in 'matching' positions on the *same* side of the transversal: 'corresponding' means 'matching'
- **Alternate angles** are between the parallel lines on *opposite* sides of the transversal: 'alternate' means 'going back and forth in turns'
- **Co-interior angles** are between the parallel lines on the *same* side of the transversal: 'co-interior' means 'together inside'

Example 2

Find the value of each variable, giving reasons.

a

b



Solution

a $y = 62$ (Alternate angles on parallel lines)

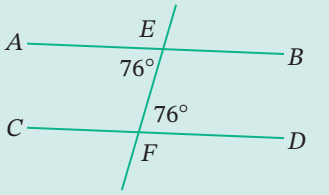
b $a + 110 = 180$ (Co-interior angles on parallel lines)

$a = 180 - 110$

$= 70$

Example 3

Prove that the lines AB and CD are parallel.



Solution

$\angle AEF$ and $\angle EFD$ are alternate angles.

$\angle AEF = \angle EFD = 76°$

$\therefore AB \parallel CD$ (Alternate angles are equal)

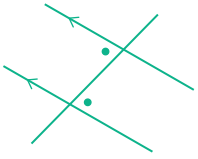
'$\therefore AB \parallel CD$' means 'Therefore line AB is parallel to line CD'

EXERCISE 4.02 ANSWERS ON P. 554

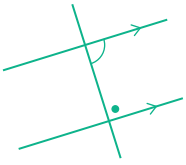
Angles on parallel lines **U F R C**

1 Is each marked pair of angles corresponding, alternate or co-interior?

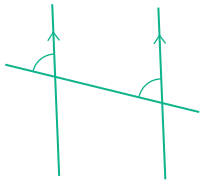
a



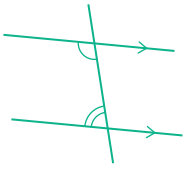
b



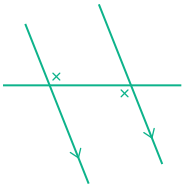
c



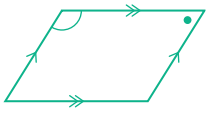
d



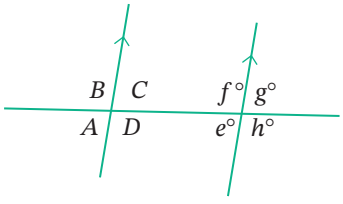
e



f



2 Which angle is corresponding to the angle marked $g°$?
Select A, B, C or D.



3 For the diagram in question **2**, name an angle that is:

a co-interior to D

b alternate to the angle marked $e°$

c equal to C

d corresponding to B

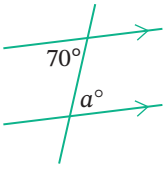
e supplementary to the angle marked $e°$.

EXAMPLE 2

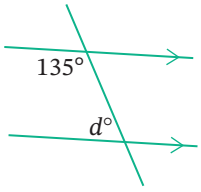
4.02

4 Find the value of the variable in each diagram, giving reasons for your answers. R C

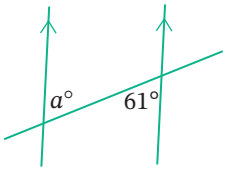
a



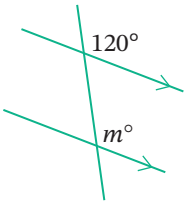
b



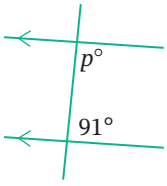
c



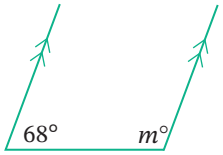
d



e

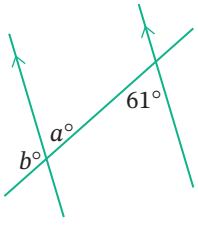


f

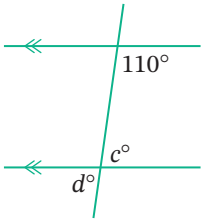


5 Find the value of each variable, giving reasons. R C

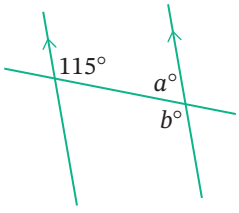
a



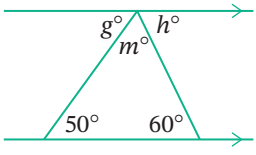
b



c



d



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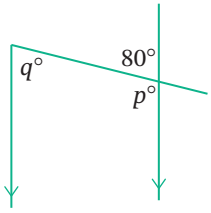

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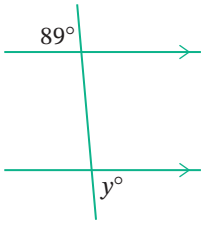
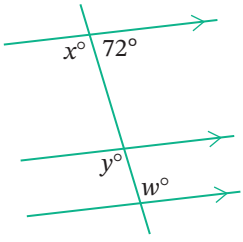
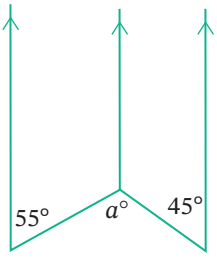


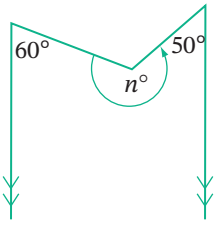
g



h



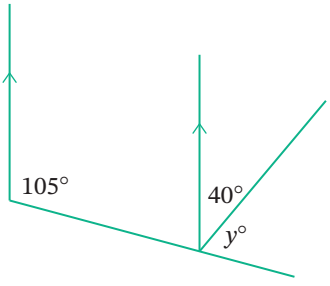
i



6 What is the value of y in this diagram? Select the correct answer **A**, **B**, **C** or **D**. R

A 85 **B** 40

C 35 **D** 65



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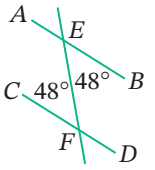
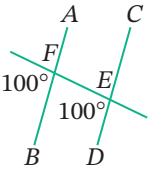
EXAMPLE 3

7 For each diagram, decide whether AB is parallel to CD. If it is, then prove it. R C

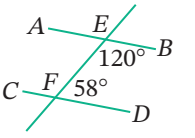
a



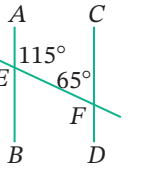
b



c

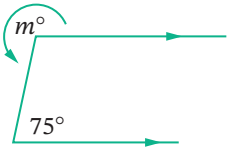


d

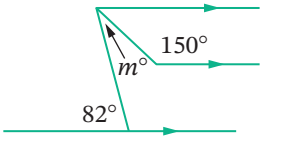


8 Find the value of m in each diagram. R

a



b



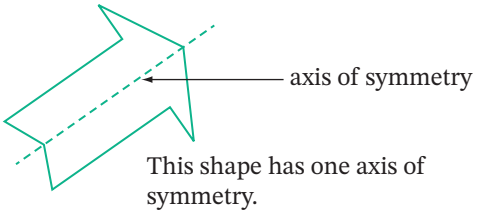
4.03 Line and rotational symmetry

Line and rotational symmetry

WS Symmetry

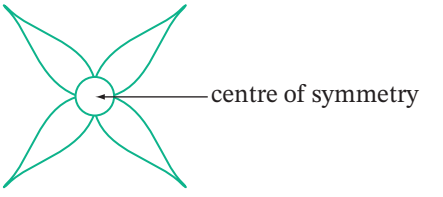
The 2 types of symmetry are **line symmetry** and **rotational symmetry**.

When a shape with **line symmetry** is folded along a line, called an **axis of symmetry**, the 2 halves of the shape fit exactly on top of each other. One half is the reflection or mirror-image of the other half.



This shape has one axis of symmetry.

When a shape with **rotational symmetry** is rotated (spun) about a point, called the **centre of symmetry**, it fits exactly on itself at least once before one full revolution (360°). The number of times the shape fits on itself in one revolution is called its **order of rotational symmetry**.



This shape has rotational symmetry of order 4.

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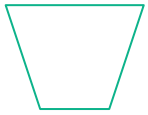
EXERCISE 4.03 ANSWERS ON P. 554

Line and rotational symmetry UFR

1 Copy each shape and mark its axes of symmetry.

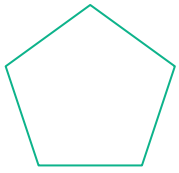
a



b

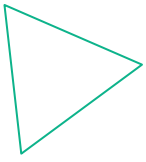
c

d



e

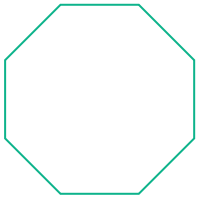
f



2 Count the number of axes of symmetry of each shape.

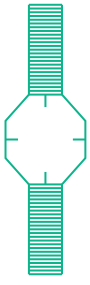
a

b



c

d



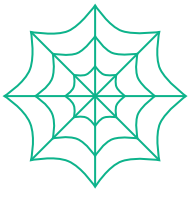
3 Decide whether each shape has rotational symmetry. If it does, state the order of rotational symmetry.

a

b

c

d



4 **a** Copy the capital letters below that have line symmetry and draw their axes of symmetry.

D H I K M N O R S V X Z

b Copy the capital letters above that have rotational symmetry and mark their centre of symmetry.

5 Draw a shape or pattern that has: R

a one axis of symmetry **b** 2 axes of symmetry **c** 4 axes of symmetry

6 Draw a shape or pattern that has rotational symmetry of order: R

a 2 **b** 4

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7 For each shape:

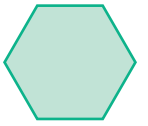
i find how many axes of symmetry it has

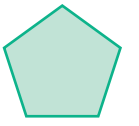
ii state whether it has rotational symmetry and if it does, state the order

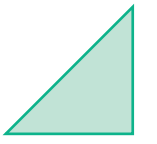
a 

b 

c 

d 

e 

f 

8 What shape has: R

a an infinite number of axes of symmetry?

b an infinite order of rotational symmetry?

4.04 Classifying triangles

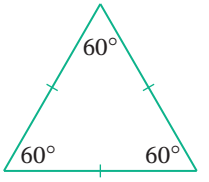
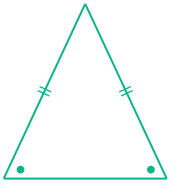
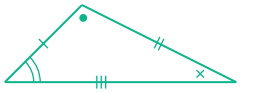
Naming shapes

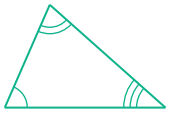
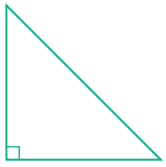
Properties of triangles

Properties of triangles

Triangle geometry

Triangles can be classified in 2 ways: by their sides or by their angles.

Classifying by sides		
Equilateral triangle	**Isosceles triangle**	**Scalene triangle**
		
3 equal sides (Also 3 equal angles, each 60°)	2 equal sides (Also 2 equal angles, opposite the equal sides)	No equal sides (Also no equal angles)

Classifying by angles		
Acute-angled triangle	**Obtuse-angled triangle**	**Right-angled triangle**
		
3 acute angles (less than 90°)	One obtuse angle (between 90° and 180°)	One right angle (90°)

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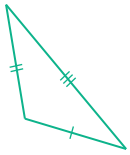
EXERCISE 4.04 ANSWERS ON P. 554

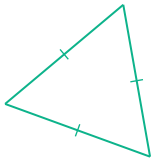
Classifying triangles UFRC

1 Classify each triangle according to its sides and angles. R C

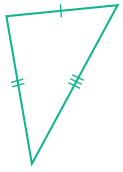
a



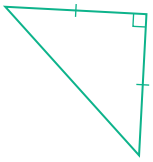
b



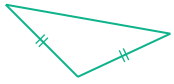
c



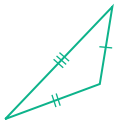
d



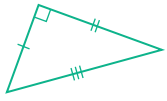
e



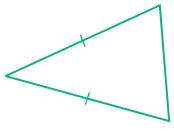
f



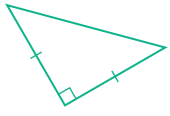
g



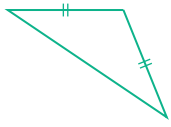
h



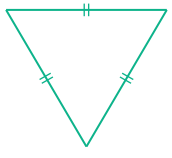
i



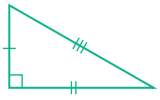
j



k



l



2 Sketch a triangle that is: R C

a right-angled and isosceles

b equilateral

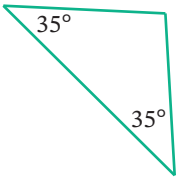
c scalene and obtuse-angled

d acute-angled and scalene

e right-angled and scalene

f acute-angled and isosceles

3 Which type of triangle is this? Select the correct answer **A**, **B**, **C** or **D**. R C



A isosceles and obtuse-angled

B isosceles and acute-angled

C scalene and obtuse-angled

D scalene and acute-angled

4 Is it possible to draw an obtuse-angled equilateral triangle? Justify your answer. R C

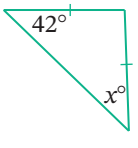
5 Which triangles in question **1** have:

a line symmetry?

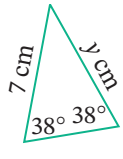
b rotational symmetry?

6 Find the value of each variable, giving a reason. R C

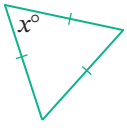
a



b

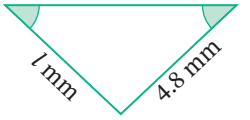


c

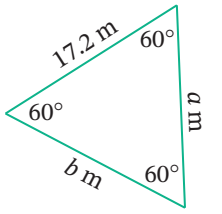


Foundation | Standard | Complex

d



e



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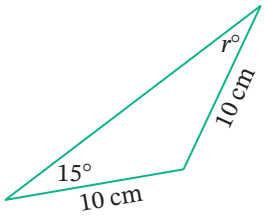


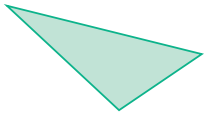
7 Is it possible to draw a triangle with 2 obtuse angles? Why? R C

8 Which triangle is both obtuse-angled and scalene? Select **A**, **B**, **C** or **D**. R C

A

B

C



D

9 Copy and complete this table. R C

Triangle	Number of axes of symmetry	Order of rotational symmetry
Equilateral triangle		
Isosceles triangle		No rotational symmetry
Scalene triangle		

Did you know?

It's all Greek or Latin to me!

Many of our words in geometry come from Greek or Latin. Latin was the language of the ancient Roman Empire.

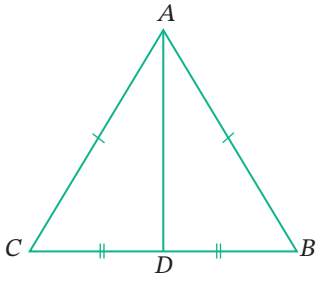
Word	Origin	Meaning
Equilateral	Latin: aequus latus	Equal sides
Equiangular	Latin: aequus angulus	Equal corners
Isosceles	Greek: isos skelos	Equal legs
Scalene	Greek: skalenos	Uneven leg
Acute	Latin: acutus	Sharp
Obtuse	Latin: obtusus	Dull or blunt
Reflex	Latin: reflexus	Bent back
Triangle	Latin: tri angulus	3 corners
Rectangle	Latin: rectus angulus	Right corners
Quadrilateral	Latin: quadri latus	4 sides
Polygon	Greek: poly gonon	Many angles
Diagonal	Greek: dia gonios	From angle to angle
Trapezium/Trapezoid	Latin/Greek: trapeza	Small table

Explain what this sentence means, and illustrate with a diagram:
'A rhombus is equilateral but not equiangular'.

Investigation

The perpendicular bisector in an isosceles triangle

$\triangle ABC$ is an isosceles triangle with $AC = AB$. It has one axis of symmetry, AD.



1 Why is $CD = DB$?

2 Why is $\angle ADC = \angle ADB$?

3 What is the size of $\angle ADC$ and $\angle ADB$?

4 AD bisects side CB. What does 'bisect' mean?

5 $AD \perp CB$. What does '$\perp$' mean?

6 'In an isosceles triangle, the axis of symmetry is the perpendicular bisector of the uneven side.' Explain what this means in your own words.

4.04

Mental skills 4A Maths without calculators ANSWERS ON P. 555

Converting fractions and decimals to percentages

To convert a fraction or decimal into a percentage, multiply it by 100%.

1 Study each example.

a $\frac{2}{5} = \frac{2}{5} \times 100\% = \frac{2}{\cancel{5}_1} \times \cancel{100}^{20}\% = 2 \times 20\% = 40\%$.

b $\frac{24}{40} = \frac{24}{40} \times 100\% = \frac{\cancel{24}^{3}}{\cancel{40}_{5}} \times 100\% = \frac{3}{\cancel{5}_1} \times \cancel{100}^{20}\% = 3 \times 20\% = 60\%$

2 Now convert each fraction to a percentage.

a $\frac{7}{10}$	**b** $\frac{33}{50}$	**c** $\frac{27}{60}$	**d** $\frac{22}{25}$	**e** $\frac{24}{32}$
f $\frac{30}{40}$	**g** $\frac{60}{75}$	**h** $\frac{4}{5}$	**i** $\frac{11}{20}$	**j** $\frac{28}{80}$
k $\frac{15}{50}$	**l** $\frac{16}{20}$	**m** $\frac{54}{60}$	**n** $\frac{18}{40}$	**o** $\frac{13}{25}$

3 Study each example.

a $0.41 = 0.41 \times 100\%$
$= 0.41$
$= 41\%$

b $0.08 = 0.08 \times 100\%$
$= 0.08$
$= 8\%$

c $0.9 = 0.9 \times 100\%$
$= 0.90$
$= 90\%$

d $0.375 = 0.375 \times 100\%$
$= 0.375$
$= 37.5\%$

4 Now convert each decimal to a percentage.

a 0.25	**b** 0.68	**c** 0.17	**d** 0.6	**e** 0.1
f 0.333	**g** 0.59	**h** 0.702	**i** 0.84	**j** 0.7
k 0.428	**l** 0.055	**m** 0.91	**n** 0.7825	**o** 0.314

4.05 Classifying quadrilaterals

Properties of quadrilaterals

Properties of quadrilaterals

Always, sometimes, never true

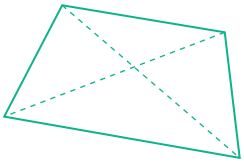
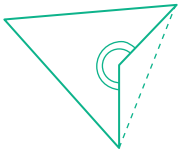
Classifying triangles and quadrilaterals

What shape am I?

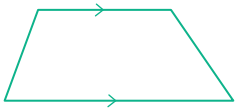
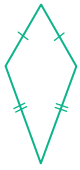
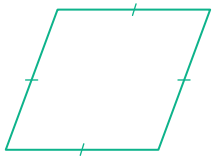
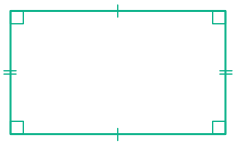
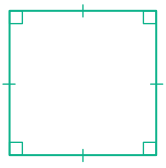
Singing in the car

Classifying quadrilaterals

A **quadrilateral** is any shape with 4 straight sides. A quadrilateral may be **convex** or non-convex.

Convex quadrilateral	Non-convex quadrilateral
	
• All vertices (corners) point outwards. • All diagonals lie within the shape. • All angles are less than 180°.	• One vertex points inwards. • One diagonal lies outside the shape. • One angle is more than 180° (reflex angle).

There are 6 special types of quadrilaterals.

Trapezium	Kite	Parallelogram
 One pair of parallel sides	 2 pairs of equal adjacent sides 'Adjacent' means 'next to each other'.	 2 pairs of parallel sides
Rhombus	**Rectangle**	**Square**
 4 equal sides	 4 right angles	 4 equal sides and 4 right angles

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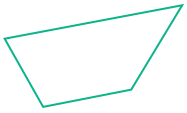
EXERCISE 4.05 ANSWERS ON P. 555

Classifying quadrilaterals U F R C

1 Is each quadrilateral convex or non-convex?

a



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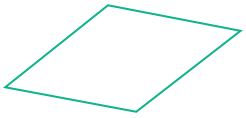
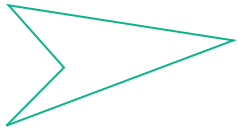
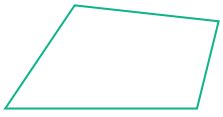


c



d

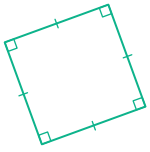
e



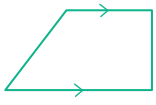
f

2 Name each quadrilateral. C

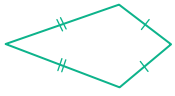
a



b



c



d

3 Name all the quadrilaterals that have: R C

a 4 right angles

b exactly one pair of parallel sides

c 4 equal sides

d opposite sides equal

e opposite sides parallel

f 2 pairs of equal adjacent sides

4 Which quadrilateral is also called a 'diamond'? R C

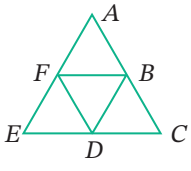
5 Copy and complete this table.

Quadrilateral	Number of axes of symmetry	Order of rotational symmetry
Rectangle		
Parallelogram		
Trapezium		No rotational symmetry
Rhombus		
Square		
Kite		

6 For this diagram, what type of quadrilateral is: C

a *ACDF*?

b *FBCD*?



Foundation | Standard | Complex

7 A parallelogram is any quadrilateral with both pairs of opposite sides parallel. Which of the following is **not** a special type of parallelogram? Select **A**, **B**, **C** or **D**. R C

A square **B** kite **C** rectangle **D** rhombus

8 Sketch each quadrilateral, showing its main features. C

a rectangle **b** trapezium **c** rhombus **d** kite

9 True or false? (Explain your answers). R C

a A rhombus is a square.

b A square is a rhombus.

c A rectangle is a parallelogram.

d A parallelogram is a quadrilateral with its opposite sides parallel and equal.

e The diagonals of a parallelogram meet at right angles.

f A square is a rectangle.

g A rectangle is a square.

Technology

Properties of quadrilaterals

In this activity, you will use your dynamic geometry software to construct quadrilaterals.

Square

1 Draw a square of length 7.5 cm.

2 Construct the 2 diagonals for the square. Measure the length of each diagonal in the square. What do you notice?

3 Now measure the **size** of each angle of the square. What do you notice?

4 Draw another square with side length 9 cm. Repeat steps 2 and 3. List 2 properties of the square.

Rectangle

1 Draw a rectangle with sides 6 cm by 3 cm.

2 Measure the length of the diagonals of the rectangle.

3 Now measure the **size** of each angle to check your accuracy. What should the size of each angle be?

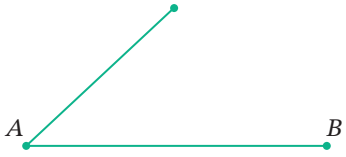
4 Draw another rectangle with length 5 cm and width 8.4 cm. Measure the length of each diagonal and the size of each angle.

5 Copy and complete this property: The _____ in a rectangle are _____.

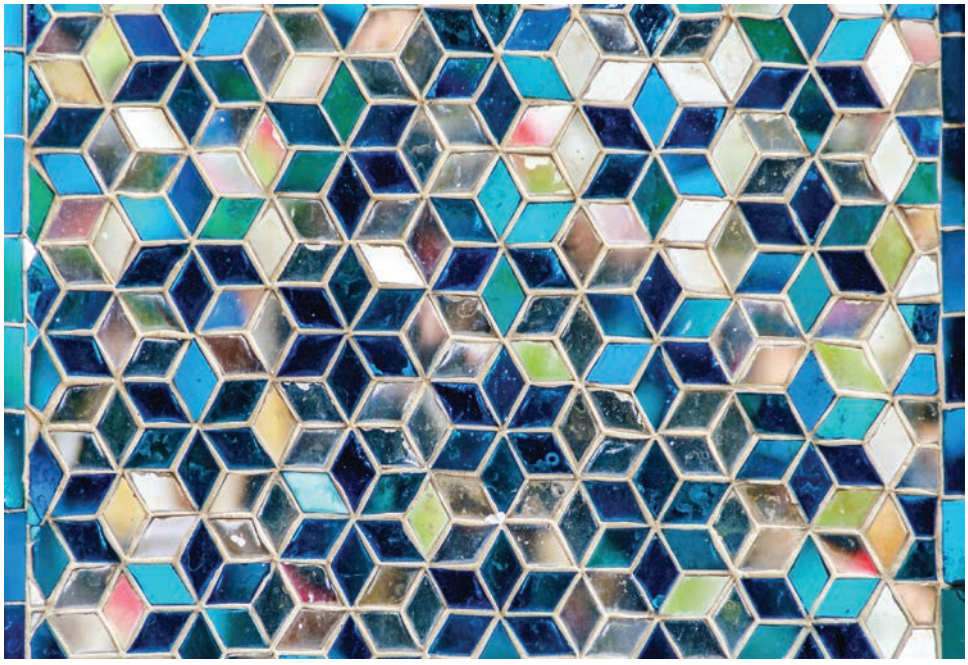
6 In one rectangle, measure the distance from each vertex to the point of intersection of the 2 diagonals. Repeat for the second rectangle. What do you notice?

7 Copy and complete this property: The _____ of a rectangle _____ each other.

Parallelogram



1. Draw an interval 6 cm long. Label it AB.
2. From A, draw the interval AC. Make the interval 4 cm.
3. Complete the parallelogram by drawing an interval from C to D and from B to D.
4. Copy and complete: The _____ sides of a parallelogram are _____.
5. Now measure the size of $\angle CAB$ and $\angle CDB$. Repeat for $\angle ACD$ and $\angle ABD$. What do you notice?
6. Copy and complete: The _____ angles of a parallelogram are _____.
7. Now draw the diagonals of the parallelogram. Measure the length of each diagonal. What do you notice?
8. Copy and complete: The diagonals of a parallelogram are _____.
9. Do the diagonals of a parallelogram bisect each other? Repeat step **6** from 'Rectangle' to help you.
10. Copy and complete: The diagonals of a parallelogram _____ bisect each other.
11. Drag any vertices of the parallelogram that you can. Is it possible to draw other parallelograms with the same dimensions, 6 cm by 4 cm? What do you notice?
12. Use your dynamic geometry software to **accurately** construct other quadrilaterals such as a rhombus, kite or trapezium.



Shutterstock.com/JoeyPhoto

Do you see rhombuses or cubes?

4.06 Properties of quadrilaterals

Properties of quadrilaterals

Properties of quadrilaterals

Diagonal properties of quadrilaterals

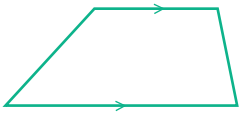
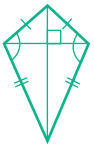
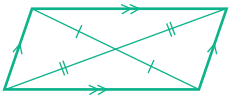
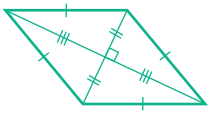
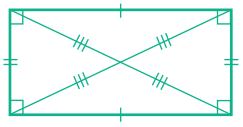
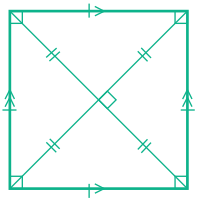
Always, sometimes, never true

Angles and shapes

EXERCISE 4.06 ANSWERS ON P. 555

Properties of quadrilaterals U F R C

1 Copy the table below or use the link to print one out. Accurately draw each of the 6 special quadrilaterals below on a sheet of paper, then cut them out. Use your shapes to help you complete the table of quadrilateral properties. R C

a	Trapezium 	• One pair of opposite sides are _____
b	Kite 	• 2 pairs of adjacent sides are _____ • One pair of opposite _____ are equal. • Diagonals intersect at _____
c	Parallelogram 	• _____ sides are equal. • Opposite _____ are parallel. • Opposite angles are _____ • Diagonals _____ each other.
d	Rhombus 	• All _____ are equal. • _____ sides are _____ • _____ angles are _____ • Diagonals bisect each other at _____ angles. • Diagonals _____ the angles of the rhombus.
e	Rectangle 	• Opposite sides are _____ • Opposite sides are also _____ • All angles are _____ • Diagonals are _____ • Diagonals _____ each other.
f	Square 	• All sides are _____ • All angles are _____ • Opposite _____ are parallel. • Diagonals are _____ • Diagonals bisect each other at _____

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2 Name all special quadrilaterals that have each property. R C

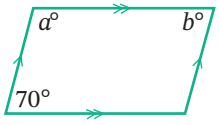
a Opposite sides are equal
b Diagonals cross at right angles
c Opposite angles are equal
d One pair of opposite sides are parallel
e Diagonals bisect each other
f Opposite sides are parallel
g Adjacent sides are of different lengths
h 2 equal diagonals
i All angles are equal
j All sides are equal

3 Copy and complete the working to find the values of a and b. R C

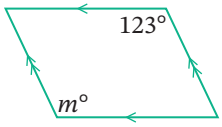
$a + 70 = 180$ (_____ angles on _____ lines)

$a =$ _____

$b =$ _____ (opposite _____ of a parallelogram)



4 Find the value of each variable, giving reasons. R C

a


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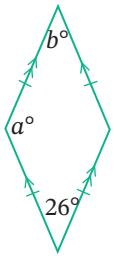

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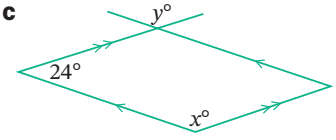

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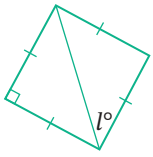

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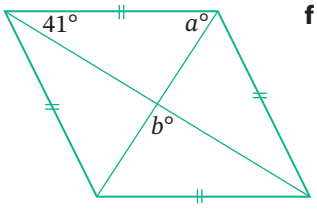

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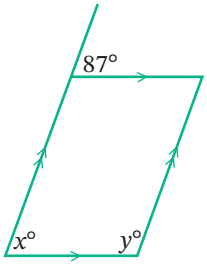

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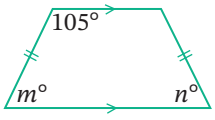

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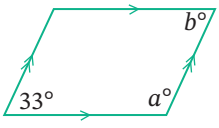

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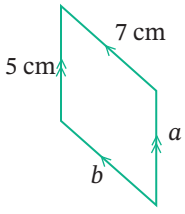

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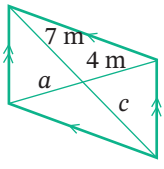
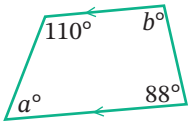


k


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5 I am a quadrilateral with opposite sides equal and parallel.

My diagonals are equal and I have 2 axes of symmetry.

Which quadrilateral am I? Select the correct answer **A**, **B**, **C** or **D**. R C

A parallelogram **B** rectangle **C** square **D** rhombus

6 I am a quadrilateral with opposite sides equal.

My diagonals bisect each other and meet at right angles.

Which quadrilateral am I? Select **A**, **B**, **C** or **D**. R C

A parallelogram **B** trapezium **C** rectangle **D** rhombus

7 A rectangle is a quadrilateral with 4 right angles.

Which one of the following is a special type of rectangle? Select **A**, **B**, **C** or **D**. R C

A square **B** kite **C** parallelogram **D** rhombus

8 A rhombus is a quadrilateral with all sides equal.

Which one of the following is a special type of rhombus? Select **A**, **B**, **C** or **D**. R C

A square **B** kite **C** parallelogram **D** rectangle

9 The quadrilateral $WXYZ$ is shown. R C

a If $WX = YZ$ and $WZ = XY$, must $WXYZ$ be a rectangle? Why?

b If $\angle WZY = 90°$, must $WXYZ$ be a rectangle? Why?

c If the information in parts **a** and **b** are both true, must $WXYZ$ be a rectangle? Why?

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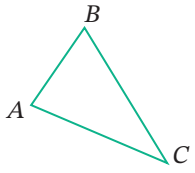
Technology

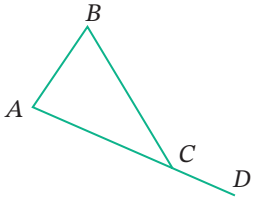
Exterior angle of a triangle

In this activity, you will use dynamic geometry software to discover an important property about the interior and exterior angles of any triangle.

1 Draw a triangle and label each vertex, A, B and C, as shown.



2 Draw a ray from A through C to point D.



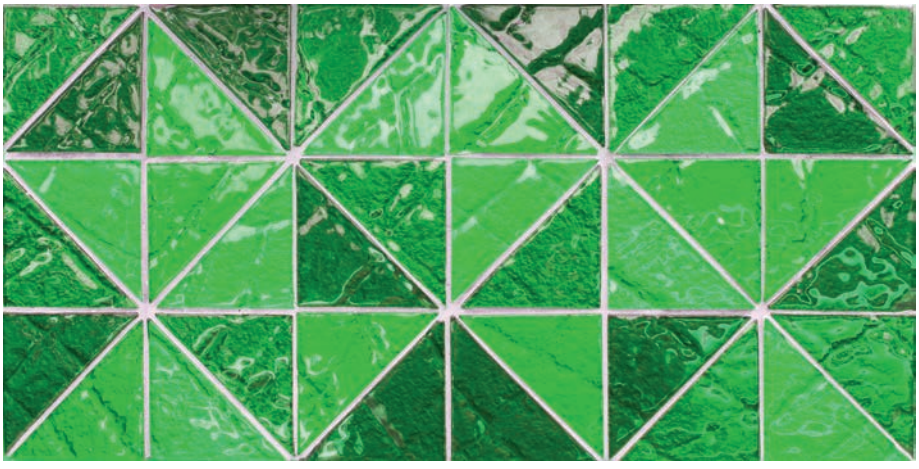
3 Find the size of $\angle ACB$, $\angle BAC$ and $\angle BCD$.

4 Calculate $\angle ACB + \angle BAC$.

5 Compare your answer from question **4** with the size of $\angle BCD$. What do you notice?

6 Repeat steps **1** to **5** for 3 other triangles.

7 Copy and complete: The _____ angle of a triangle is _____ to the sum of the _____ opposite _____ angles.



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4.07 Angle sums of triangles and quadrilaterals

Angle sums of triangles and quadrilaterals

Angle in polygons

WS Angles in triangles

WS Find the unknown angle 2

Mixed angles

Angles in triangles

Triangles and quadrilaterals

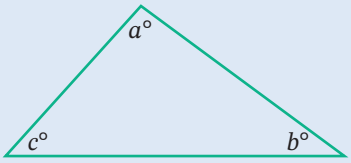
WS Deductive geometry

WS Shapes and angles review

Angle sum of a triangle

The angle sum of a triangle is 180°.

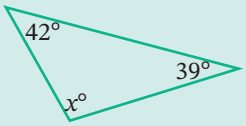
$$a + b + c = 180$$



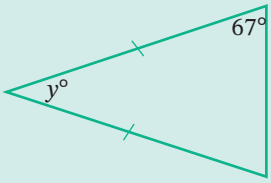
Example 4

Find the value of each variable, giving reasons.

a



b



Solution

a $x + 42 + 39 = 180$ (angle sum of a triangle)

$x = 180 - 42 - 39$

$= 99$

b $y + 67 + 67 = 180$ (angle sum of an isosceles triangle)

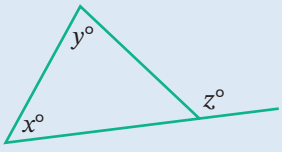
$y = 180 - 134$

$= 46$

Exterior angle of a triangle

The exterior angle of a triangle is equal to the sum of the 2 interior opposite angles.

$$z = x + y$$



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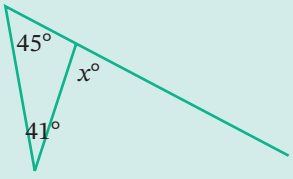
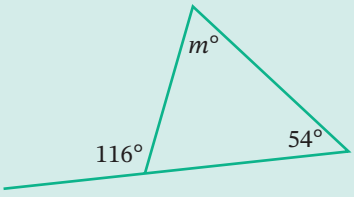
Example 5

Find the value of each variable, giving reasons.

a



b



Solution

a $x = 45 + 41$ (exterior angle of a triangle)

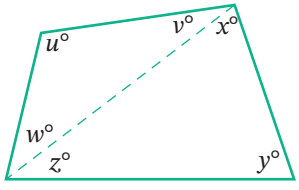
$= 86$

b $m + 54 = 116$ (exterior angle of a triangle)

$m = 116 - 54$

$= 62$

To prove the angle sum of a quadrilateral, we can divide a quadrilateral into 2 triangles along one of its diagonals. Because the angles in each triangle add to 180°, the angles in both triangles add to $2 \times 180° = 360°$.



$u° + v° + w° = 180°$ and $x° + y° + z° = 180°$

$\therefore$ Angle sum of quadrilateral $= 180° + 180°$

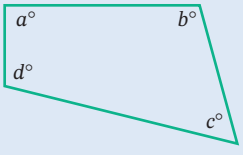
$= 360°$

Angle sum of a quadrilateral

The angle sum of a quadrilateral is 360°.

$$a + b + c + d = 360$$

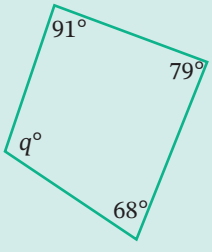
This property is true for both convex and non-convex quadrilaterals.



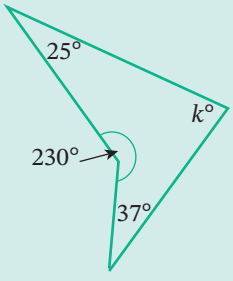
Example 6

Find the value of each variable, giving reasons.

a



b



Solution

a $q + 91 + 79 + 68 = 360$ (angle sum of a quadrilateral)

$q + 238 = 360$

$q = 360 - 238$

$= 122$

b $k + 25 + 37 + 230 = 360$ (angle sum of a quadrilateral)

$k + 292 = 360$

$k = 360 - 292$

$= 68$

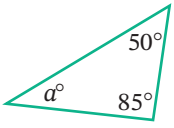
EXERCISE 4.07 ANSWERS ON P. 556

Angle sums of triangles and quadrilaterals U F R C

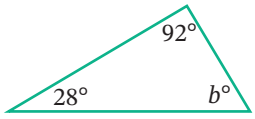
EXAMPLE 4

1 Find the value of each variable. R

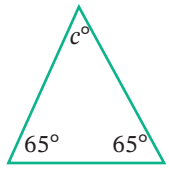
a



b



c



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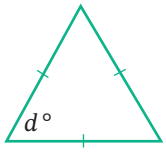

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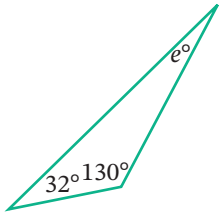

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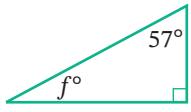

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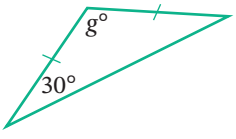

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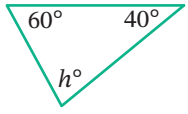

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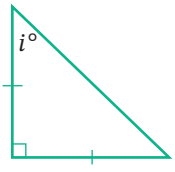
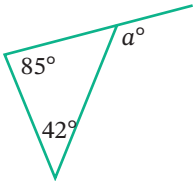


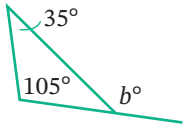
EXAMPLE 5

2 Find the value of each variable. R

a



b



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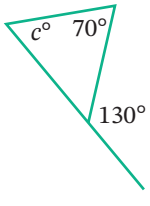

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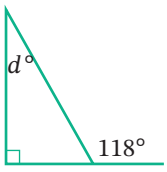
d



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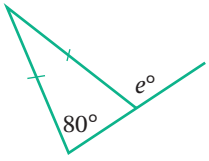

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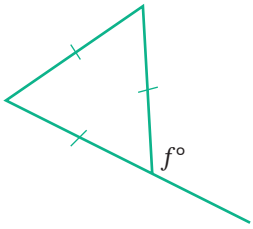
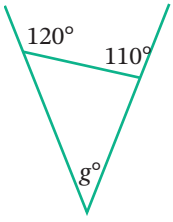


g



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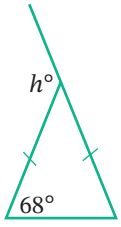

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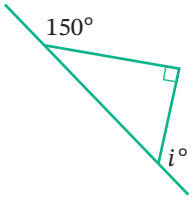
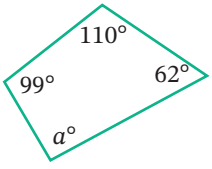


3 Find the value of each variable. R

EXAMPLE 6

a



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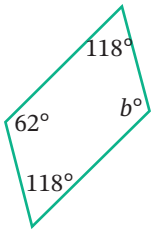
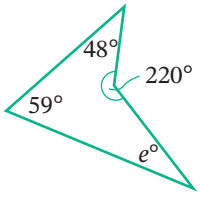


c

d

e



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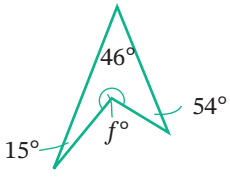

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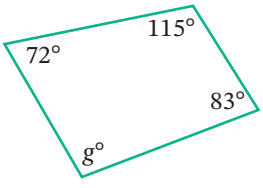

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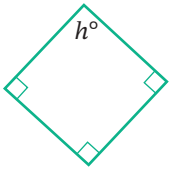

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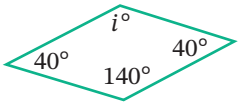
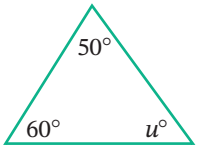


4 Find the value of u. Select the correct answer **A**, **B**, **C** or **D**.

A 50 **B** 60 **C** 70 **D** 80



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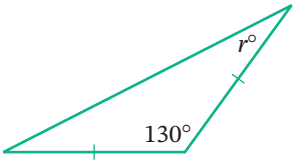
5 Find the value of r. Select **A**, **B**, **C** or **D**.

A 25 **B** 35

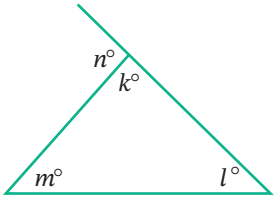
C 50 **D** 100



6 Which equation is correct for this triangle? Select **A**, **B** or **C**. R

A $n = l + m$ **B** $n = k + m$

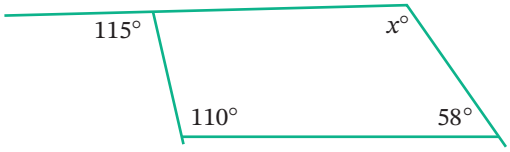
C $n = l + k$



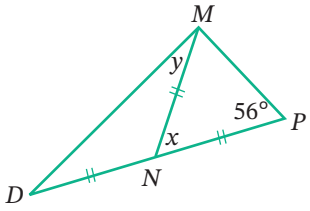
7 Find the value of x. Select **A**, **B**, **C** or **D**.

A 115 **B** 65

C 127 **D** 122



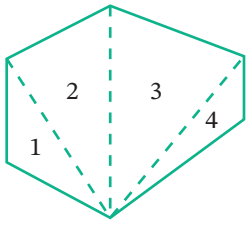
8 Find the size of angles x and y, giving reasons. R C



Investigation

Angle sum of a polygon

We know that the angle sum of a triangle is 180° and that the angle sum of a quadrilateral is 360°, but how do we find the angle sum of other convex polygons?

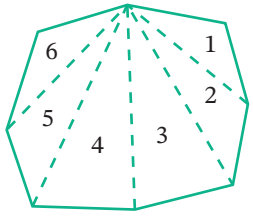


A hexagon can be divided into 4 triangles by drawing the diagonals from one vertex.

The sum of the angles in a hexagon = (angle sum of a triangle) × 4

$= 180° \times 4$

$= 720°$



An octagon can be divided into 6 triangles by drawing the diagonals from one vertex.

The total of the angles in an octagon = $180° \times 6 = 1080°$.

1 How is the number of sides related to the number of triangles formed in a shape?

2 Copy and complete these sentences:

The angle sum of a polygon with n sides is

$A = 180 \times (\text{number of sides} - \_\_\_\_\_)°$

or: $A = 180 \times (n - \_\_\_\_\_)°$

3 Find the angle sum of each polygon.

a 11-sided polygon **b** 20-sided polygon **c** 14-sided polygon

Extension: Angle sum of a polygon 4.08

YEAR 9 STAGE 5.2

WS Angle sum of a polygon

WS Angles in regular polygons

A **convex polygon** has all vertices pointing outwards.

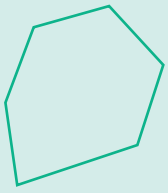
A **regular polygon** has all sides the same length and all angles the same size.

Example 7

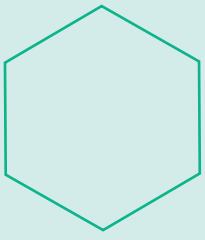
These shapes are all hexagons (6-sided).

A

B



C



a Which hexagons are convex?

b Which hexagon is regular?

Solution

a Hexagons **B** and **C** are convex because all of their vertices point outwards.

b Hexagon **C** is regular, because all its sides are equal and all its angles are equal.

Angle sum of a polygon

The angle sum of a polygon with n sides is given by the formula:

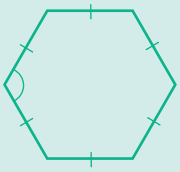
$$A = 180(n - 2)°$$

This property applies to both convex and non-convex polygons.

YEAR 9
STAGE 5.2

Example 8

Find the size of one angle in a regular hexagon.



Solution

A hexagon has 6 sides ($n = 6$).

Angle sum of a hexagon $= 180(6 - 2)°$

$= 180 \times 4°$

$= 720°$

For a regular hexagon:

One angle $= 720° \div 6$

$= 120°$

$\therefore$ Each angle in a regular hexagon is 120°.

EXERCISE 4.08 ANSWERS ON P. 556

Extension: Angle sum of a polygon U F R C

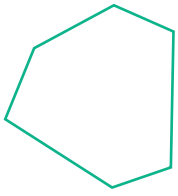
EXAMPLE 7

1 Copy each polygon and write its correct name from this list.

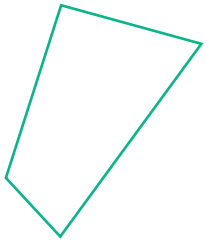
hexagon	nonagon	heptagon	decagon
octagon	triangle	quadrilateral	pentagon

Also, state if each polygon is **regular** or **irregular**, **convex** or **non-convex**. C

a



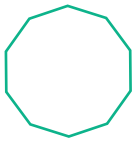
b



c

d

e



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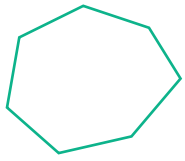

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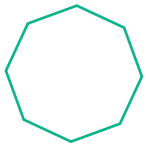

h

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2 Copy and complete this table. C

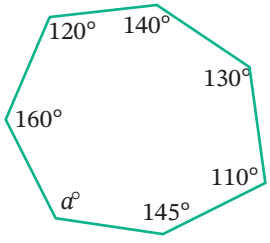
Polygon	Number of sides	Sum of angles inside polygon
hexagon		
heptagon		
octagon		
nonagon		
decagon		

3 Find the sum of the interior angles in:

a a 15-agon **b** a 20-agon **c** a 25-agon **d** a 100-agon

4 Find the value of each variable. R

a



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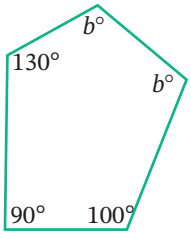

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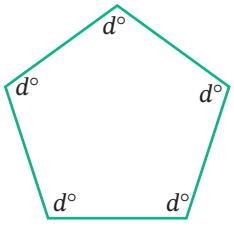

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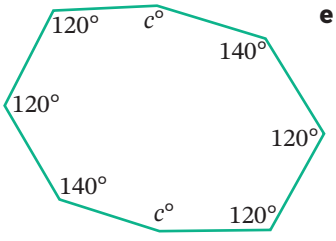

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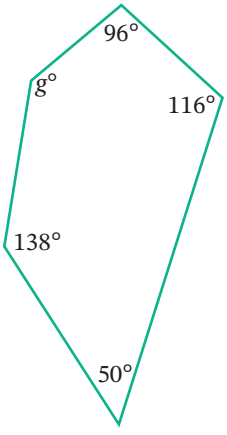

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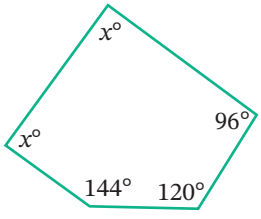

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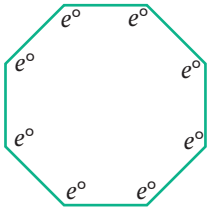

5 Find the size of one interior angle in each regular polygon.

EXAMPLE 8

a square **b** equilateral triangle **c** regular hexagon

d regular octagon **e** regular decagon **f** regular pentagon

g regular dodecagon

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6 Find the number of sides of the polygon that has an angle sum of: R

a 2160° **b** 5760° **c** 4320° **d** 9180° **e** 22 140°

7 Is it possible for a regular polygon to have each interior angle:

a 144°? **b** 130°?

If so, find the number of sides. R

Mental skills 4B Maths without calculators ANSWERS ON P. 556

Converting decimals and percentages to fractions

1 Consider each example.

a $0.35 = \frac{35^{7}}{100_{20}} = \frac{7}{20}$

(2 decimal places, two 0s in the denominator)

b $0.8 = \frac{8^{4}}{10_{5}} = \frac{4}{5}$

(one decimal place, one 0 in the denominator)

c $0.64 = \frac{64^{16}}{100_{25}} = \frac{16}{25}$

d $0.22 = \frac{22^{11}}{100_{50}} = \frac{11}{50}$

2 Now convert each decimal to a fraction.

a 0.75 **b** 0.28 **c** 0.3 **d** 0.14

e 0.06 **f** 0.85 **g** 0.32 **h** 0.49

i 0.56 **j** 0.9 **k** 0.72 **l** 0.65

m 0.2 **n** 0.24 **o** 0.53

3 Consider each example.

a $26\% = \frac{26^{13}}{100_{50}} = \frac{13}{50}$

b $40\% = \frac{40^{2}}{100_{5}} = \frac{2}{5}$

c $8\% = \frac{8^{2}}{100_{25}} = \frac{2}{25}$

d $95\% = \frac{95^{19}}{100_{20}} = \frac{19}{20}$

4 Now convert each percentage to a fraction.

a 76% **b** 10% **c** 80% **d** 45%

e 88% **f** 56% **g** 75% **h** 31%

i 68% **j** 5% **k** 60% **l** 54%

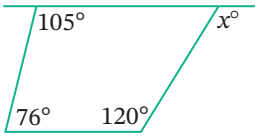
m 6% **n** 49% **o** 82%

Power plus ANSWERS ON P. 556

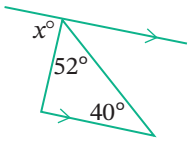
As questions become more complex, it may not be possible to find the answer in one step. It may be necessary to find another angle first.

Find the value of x in each diagram. (Give reasons for all steps.)

a



b



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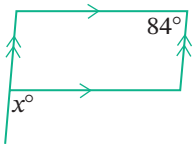

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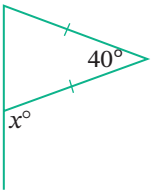

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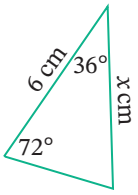

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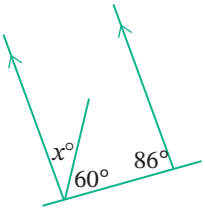

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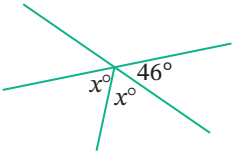

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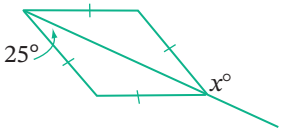

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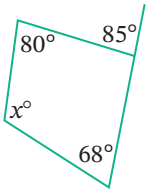

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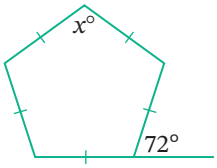

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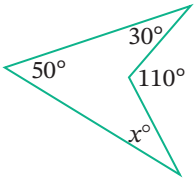

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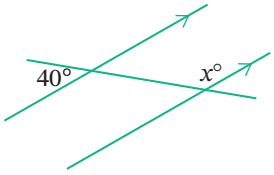


Shapes puzzle 1

Equal angles

4.08

Shapes and angles

CHAPTER 4 REVIEW

Language of maths

acute	alternate	angle sum	axis/axes
co-interior	complementary	convex	corresponding
diagonal	equilateral	exterior angle	interior angle
isosceles	kite	obtuse	parallelogram
quadrilateral	reflex	rhombus	rotational symmetry
scalene	supplementary	trapezium	vertically opposite

1 What are **supplementary angles**?

2 Name the types of angles associated with parallel lines cut by a transversal.

3 What type of triangle has one angle that is greater than 90°?

4 What is the **angle sum** of a quadrilateral?

5 Illustrate the difference between an **interior angle** and an **exterior angle** of a triangle.

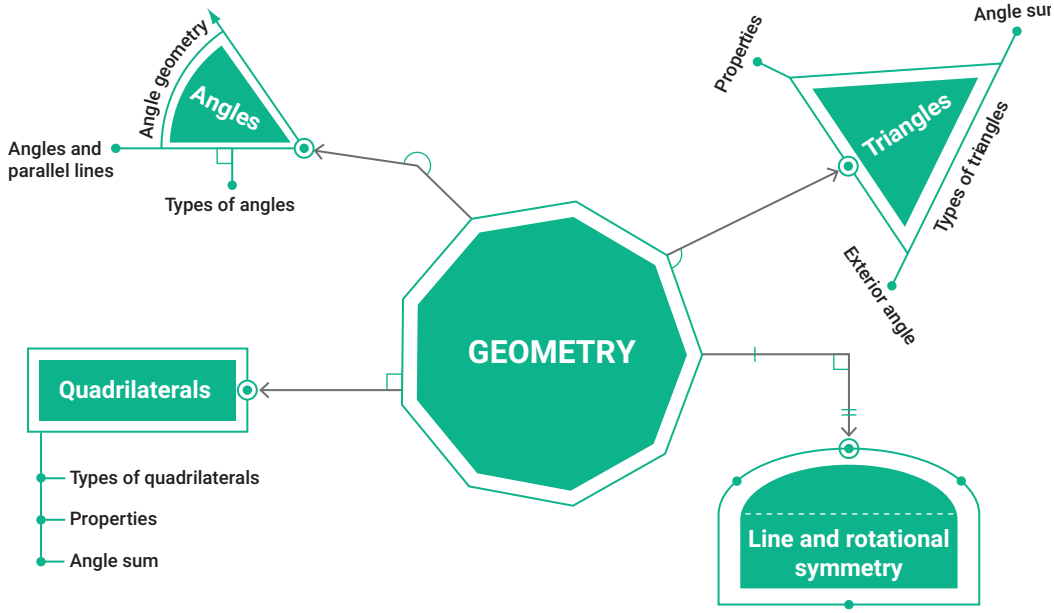
6 What does **equilateral** mean? What is the common name for an 'equilateral parallelogram'?

Mind map: Geometry

Topic summary

- Do you think this chapter is useful? Why? What did you learn in this chapter?
- How confident do you feel with geometry?
- List anything in this chapter that you did not understand. Show your teacher.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



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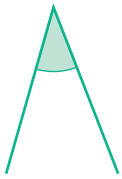
TEST YOURSELF 4

ANSWERS ON P. 556

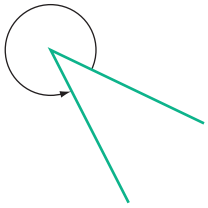
1 Name each type of angle. 4.01

a

b

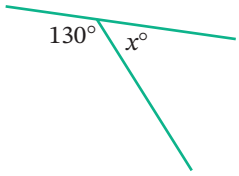


c

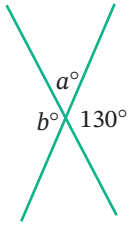


2 Find the value of each variable, giving reasons. 4.01

a



b



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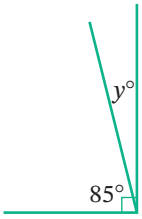
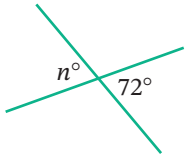
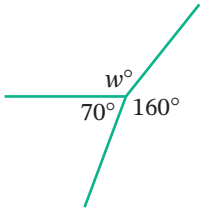


d

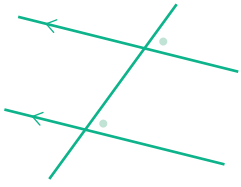


e

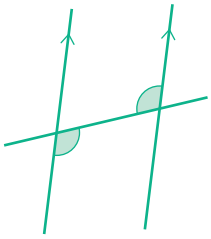


3 Name each type of angle pair. 4.02

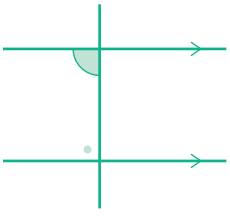
a



b

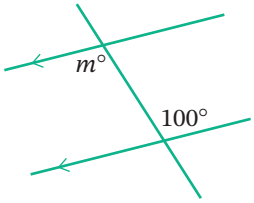


c



4 Find the value of each variable, giving reasons. 4.02

a



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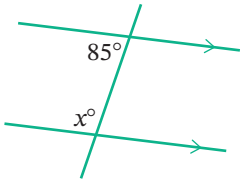

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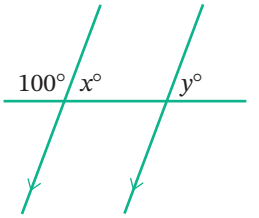
c



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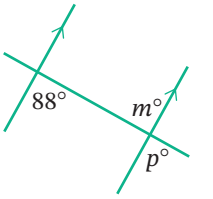
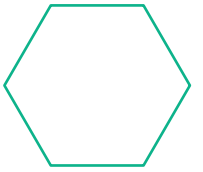


4.03

5 Copy each shape and mark in its axes of symmetry.

a



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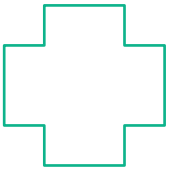


c

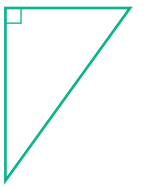
4.03

6 For each shape above, decide whether or not it has rotational symmetry. If it does, state the order of rotational symmetry.

4.04

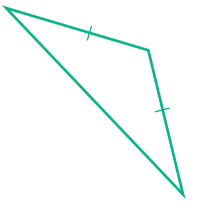
7 Classify each triangle by its sides and angles.

a



b

c



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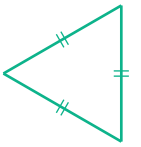

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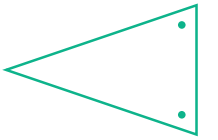

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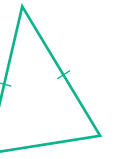
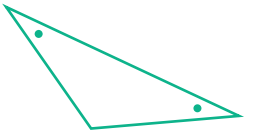


g

h

i



4.05

8 Sketch each quadrilateral.

a parallelogram **b** kite **c** trapezium

4.06

9 Name all the quadrilaterals that have each property.

a All sides equal in length

b No parallel sides

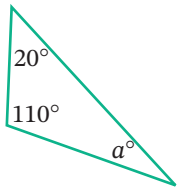
c All angles right angles

d 2 pairs of opposite angles equal

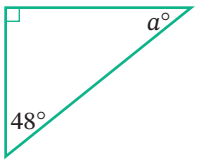
e Diagonals bisect each other

10 Find the value of each variable. 4.07

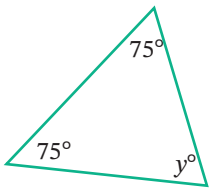
a



b



c



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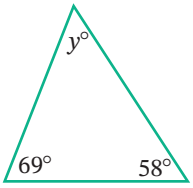
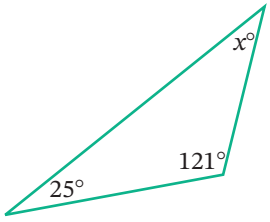


e



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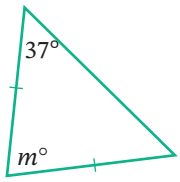
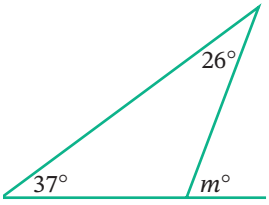
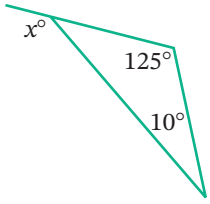


11 Find the value of each variable. 4.07

a



b



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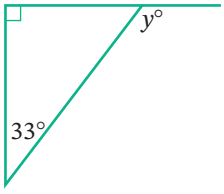
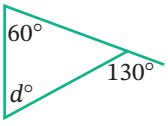


d



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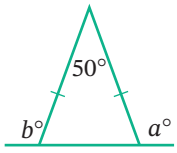

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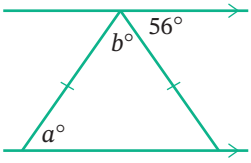

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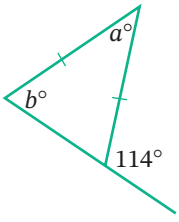
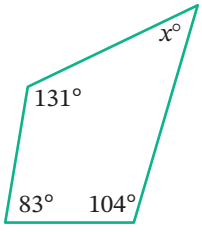
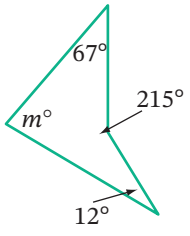


12 Find the value of each variable. 4.07

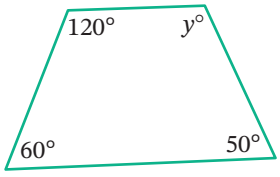
a



b



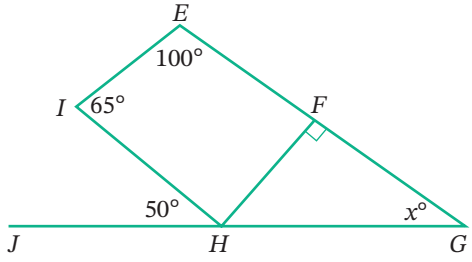
c



13 Find the value of x. Select **A**, **B**, **C** or **D**. 4.07

A 65 **B** 25

C 55 **D** 80



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